0965-8564(94)00030-1

TRANSIT CENTRE LOCATION- ALLOCATION DECISIONS

DEAN H. UYENO

Management Science Division, Faculty of Commerce, The University of British Columbia,
Vancouver, Canada, V6T 1Z2

KEITH A. WILLOUGHBY

Faculty of Management, The University of Calgary, Calgary, Canada, T2N 1N4

(Received 25 August 1993; in revised form 6 June 1994)

Abstract—A computationally feasible methodology to locate, size, and determine the number of transit centres (bus barns or garages) for an urban transit system is described. This methodology, known by the acronym BUBLS (BUs Barn Location System), is applied to a case study with BC Transit, a large Canadian urban transit system. As the case study illustrates, BUBLS is currently employed in capital projects planning.

1. INTRODUCTION

There are substantial nonproductive activities and costs in a transit system.

A bus does not commence revenue service until it “deadheads” from the transit centre (bus barn or garage) at which it is housed to an initiation point along its assigned route. Likewise, at the end of revenue service, a bus “deadheads” back to its transit centre. The sum of these two travel times constitutes *vehicle deadhead time*, a costly activity. Deadhead costs should be balanced off against the capital costs of more transit centres.

An additional cost is incurred on a bus “run” (a vehicle service schedule) that extends longer than the maximum driver time allowed by work rules, often established by union contract. When a relief driver takes control of the bus, the former driver may be paid for the time required to return to the point where his/her shift began. This time constitutes *driver relief time*. Whereas this costs less than vehicle deadhead costs, nevertheless the costs are substantial.

As urban areas grow in population and/or the spatial distribution of transit customers changes, even the best-chosen transit centre locations may become less efficient. It is difficult to obtain funding for proper analysis because most transit systems operate with substantial chronic deficits. Management may choose to use their resources for more visible actions, such as purchasing additional buses. Consequently, inefficiencies can grow unobtrusively. An existing transit centre may strain at the seams as more buses are added to an already overcrowded facility. Buses may incur greater deadhead costs as they travel between their assigned transit centre and their route. Large sums of money are involved.

Typically, a thorough analysis is triggered only when a major change in a transit system is under consideration. That was the case for this study. A methodology to determine the appropriate number, location, and size of transit centres was created. This took into account advances in hardware and software since previously-published studies. This was then successfully applied to the current network of bus routes and transit centres for this large urban transit system. It is presently being used to evaluate the effects of new advanced light rapid transit routes. A literature review, the methodology, and the case study are discussed.

2. LITERATURE REVIEW

The determination of the optimal number, location, and size of transit centres is one of a class of problems known as *location/allocation problems*. It is one of the oldest classes of problems tackled in Management Science (see Balinski, 1961; Cooper, 1963).

Warehouses, ambulance centres, and audit offices are but a few situations in which analytic techniques for location/allocation analysis have been used (see Love et al., 1988).

Mixed integer programming is one method used to solve these problems (see Bent, 1972). A least cost solution can be derived that ensures the capacity of locations is not violated and the demand for bus service is satisfied. However, computational times can grow very long as the number of integer variables in the problem formulation increases.

A number of transit systems have attempted to use location/allocation analysis. Perhaps the most successful application has been the work of Thomas H. Maze. Maze et al. (1982, 1983) created a mixed integer programming formulation for the problem of locating and sizing transit centres. Their formulation sought to minimize vehicle dead-head and driver relief costs as well as transit centre operating and capital costs. They used a 0-1 variable to assign a "block" (route assignment of a bus) to a transit centre.

Given the computer resources of the day, it was impossible for Maze to solve the problem as formulated. A transit system with 1800 blocks of bus service and 10 existing or potential transit centres would generate 18,000 assignment variables. However, he adjusted his formulation by dropping the fixed charge ("switch" or 0-1) variables for opening/not opening a site. The transit centre location problem was then solved as a series of transportation problems, a type of problem which has a naturally occurring integer solution. Transit centres were excluded or included in subsequent solutions through applications of Khumawala's delta and omega heuristics (see Khumawala, 1974). These heuristic decision rules compared the incremental variable transportation costs against the fixed cost of constructing a facility. Maze and his group were then able to successfully apply this methodology to a transit system in Detroit, MI.

3. MODEL FORMULATION

For BC Transit, a mixed integer programming model was formulated to minimize costs for the appropriate number, location, and size of transit centres. This approach considers the existing configuration of bus routes as well as a set of existing and candidate transit centre locations. Current workstation hardware and mathematical programming software enable this particular problem to be formulated and solved expeditiously. The elements of the formulation are discussed in the rest of Section 3.

3.1. The mathematical objective function

Minimize total cost =

$$\sum_r \sum_d \sum_p \sum_s C_{rdps} X_{rdps} + \sum_{s\text{-candidate}} V_s N_s + \sum_{s\text{-BTC}} F_s W_s - \sum_{s\text{-existing}} R_s Z_s$$

where:

The subscripts:

- r route
- d day
- p service period
- s transit centre

C_{rdps} = the annual deadheading cost to operate one bus on route r on day d for service period p from transit centre s .

X_{rdps} = the number of buses assigned to route r on day d for service period p from transit centre s .

V_s = annualized per bus capital costs to construct a candidate transit centre at site s .

N_s = the total number of buses assigned to transit centre s (including spares).

F_{BTC} = annualized capital costs to electrify Boundary Road between Hastings Street and Broadway so that trolleys could be allocated to the Burnaby Transit Centre (BTC).

- $W_{BTC} = 0$ if Boundary Road is not electrified between Hastings and Broadway (trolleys are then not assigned to BTC).
- 1 Boundary Road is electrified between Hastings and Broadway (trolleys are allocated to BTC).
- (Because this variable and its cost coefficient only apply to one transit centre location, the subscript is given as "BTC," instead of "s").
- R_s = annualized "salvage value" to eliminate an existing transit centre s .
- Z_s = 0 if *existing* transit centre s remains open.
1 if *existing* transit centre s is shut down.

The objective function sums the total location/allocation costs.

The first term in the objective function is the vehicle deadhead cost. This is determined for each route-day-service period-transit centre combination (subscripted "rdps"). Days are broken down into three categories: Monday-Friday, Saturday, and Sunday-holiday categories because the total number of buses required per route will vary according to these categories. Service periods are composed of a.m. peak runs, p.m. peak runs, and two types of all-day runs (according to duration). This breakdown was used because higher driver relief costs are incurred on the longer runs. Later, the effect of the optimal solution on driver relief is discussed.

The second term in the objective function calculates the total capital cost required to construct candidate transit centres.

The third term in the objective function is used to assign a capital cost should the Burnaby Transit Centre (BTC) be modified to accept trolley buses. BC Transit's bus system in metropolitan Vancouver consists of both diesel and trolley buses. Because trolley buses use overhead electrical wires, they are restricted in the routes they can serve and the transit centres to which they can be assigned.

The last term considers the salvage value that would accrue to BC Transit should an existing transit centre be closed.

3.2. Constraints

Each class of constraints is now discussed.

3.2.1. "Demand" constraint.

$$\sum_s X_{rdps} = D_{rdp} \quad \forall r, d, p$$

The formulation of this key constraint was suggested by Derek Atkins (1992). It forces the total number of assigned buses for a given route on a certain day for a specific service period to equal the demand. The demand data for each combination was readily available from the Scheduling Department's run schedules.

The mixed integer model proposed by Maze used binary integer variables to assign bus "blocks" to transit centres. As explained earlier, this resulted in a large number of integer variables. In the BC Transit system with 1899 runs and ten transit centre locations to analyze, this would have resulted in almost 19,000 integer (assignment) variables.

The constraint above transforms the assignment of buses to transit centres by utilizing continuous assignment variables. Consider a route operating during the a.m. peak on a day that requires 11 buses. In this case, the demand, D_{rdp} , for that combination would be set to 11. The X_{rdps} variable is summed over all sites and forced to equal this demand figure. Maze's formulation would have required 11 constraints to handle this scenario; this formulation uses one. However, a price is paid for the reduction in the number of integer variables. Potentially, the buses operating a given route could be split between alternative transit centre locations. In other words, the demand figure, D_{rdp} , could be spread over two (or more) bus garages.

A further benefit of this constraint is its ability to model alternative planning scenarios. Suppose personnel from the Planning Department feel that service should increase on a certain route on a certain day for a certain service period. Then, the D_{rdp} numbers for that combination can be increased to reflect this conjecture.

3.2.2. Allocation of buses to transit centres (active buses).

$$\sum_r \sum_d \sum_p X_{rdps} - A_s \leq 0 \quad \forall s$$

This constraint sums the total number of buses assigned to a specific transit centre and ensures that it does not exceed A_s , the active number of buses assigned to a transit centre. Because a bus operating in the a.m. peak can also operate in the p.m. peak, a depot's active number of buses is the maximum of the all-day plus the a.m. peak runs or all-day plus p.m. peak runs. The summation is done for the three different day periods considered (Monday–Friday, Saturday, and Sunday–holiday).

3.2.3. Allocation of buses to transit centres (total buses).

$$N_s - \alpha A_s \geq 0 \quad \forall s$$

In these constraints, the total number of buses allocated to each transit centre is determined. The spare factor, α , is used to augment the number of active buses to allow for vehicle breakdowns, major scheduled maintenance, or accidents. The same spare factor was used for both trolleys and diesels.

3.2.4. Candidate transit centre locations.

$$N_s - \omega_s Y_s \leq 0 \quad \forall s = \text{candidate}$$

$$N_s - \lambda_s Y_s \geq 0 \quad \forall s = \text{candidate}$$

where:

$Y_s = 0$ if a transit centre in candidate location s is not opened

1 if a transit centre in candidate location s is opened

$\omega_s =$ maximum allowed size of candidate transit centre s

$\lambda_s =$ minimum allowed size of candidate transit centre s

The sizes of the candidate transit centres were forced to fall in pre-assigned capacity bounds. This reflected the judgments of transit staff concerning the appropriate values for minimum and maximum sizes of transit centres.

3.2.5. Existing transit centres locations.

$$N_s - \beta_s (1 - Z_s) \leq 0 \quad \forall s = \text{existing}$$

$$N_s - \gamma_s (1 - Z_s) \geq 0 \quad \forall s = \text{existing}$$

where

Z_s is as described previously

$\beta_s =$ maximum allowed size of existing transit centre s

$\gamma_s =$ minimum allowed size of existing transit centre s

As with the candidate depots, the size of existing transit centres is forced to fall within certain bounds. This eliminated the situation where an optimal solution might keep open an existing transit centre and assign it, say, 12 buses.

3.2.6. Electrification for Burnaby Transit Centre.

$$\sum_{r=\text{trolley}} X_{rdpBTC} - \beta_{BTC} W_{BTC} \leq 0 \quad \forall d, p$$

Should any trolleys be assigned to the Burnaby Transit Centre, this constraint insures that the annualized capital cost of the necessary electrification is considered in the objective function by setting W_{BTC} to 1.

3.2.7. Other constraints.

Variable restrictions:

$$\begin{aligned} X_{rdps} &\geq 0 \\ A_s, N_s &\geq 0 \text{ and integer} \\ \omega_s, \lambda_s, \beta_s, \gamma_s, \theta_s &\geq 0 \\ \alpha_s &\geq 1 \\ W_s, Y_s, Z_s &= 0, 1 \end{aligned}$$

Each of the 10 existing and candidate transit centre sites required a general integer variable for busses required for service, A_s , and for busses required including spares, N_s . Binary integer (0,1) variables were used to indicate electrification of the Burnaby Transit Centre (W_s), opening of 1 of the 5 candidate transit centres (Y_s), and continued operation of 1 of the 5 existing transit centres (Z_s). Thus, this formulation produced a mixed integer program with only 31 integer variables (20 general and 11 binary). As was previously mentioned, this reduction in the number of integer variables was made possible through the demand constraint formulation described in Section 3.2.1.

3.3. Relevant costs

3.3.1. Travel costs. The determination of deadhead travel costs first required the estimation of travel time from each transit centre to the initiation and termination points. Currently, BC Transit uses visual estimates and prior experience to predict vehicle deadhead time.

For this study, linear regression was employed to estimate appropriate deadhead travel times. A subset of 40 transit centre-deadhead point pairs were chosen. The actual deadhead time allowed on the BC Transit run schedules for these origin-destination pairs was then obtained. The number of kilometres for each of 5 road types (from expressways to congested urban streets) was determined between each deadhead point-transit centre pair. A linear regression of deadhead travel time against the number of kilometres by road type variable was performed. This produced a set of regression coefficients for travel time/km for each road type. For each route-transit centre combination, multiplying the road distances by the corresponding regression coefficients and summing the results gave the vehicle deadhead time. This could then be multiplied by the hourly vehicle deadhead cost to produce a total deadhead cost for each route-transit centre combination.

Examination of the actual deadhead points used by BC Transit revealed that a substantial portion were set up to serve the existing transit centre location scheme. In an effort to make the deadhead times independent of transit centre location, the termini of the routes were used as the initiation/termination points.

3.3.2. Capital costs. Estimates of the capital costs to construct candidate transit centres were obtained from the Capital Projects and Planning Departments at BC Transit. The per bus capital cost (exclusive of land) was obtained from BC Transit's most recent transit centre construction. Land costs varied greatly by transit centre location. A space requirement of 25 buses per acre was assumed in the determination of the per bus capital charge. After extensive discussion with BC Transit staff, linear construction costs were assumed within the range of facility design size.

Salvage values for existing transit centres were determined by calculating annualized land value, based on data supplied by local real estate agencies. Values accruing from the resulting sale of equipment were not considered.

3.3.3. Operation costs. Maze's formulation included the operating costs of transit centre locations. However, this formulation excludes them. Discussions with representatives of the Capital Projects Department of BC Transit revealed that facility size had little impact on marginal operating costs.

4. BC TRANSIT CASE STUDY

This methodology was tested on the Vancouver Regional Transit System of BC Transit. This system serves the largest transit service area in Canada, covering an area roughly 1800 km². Its system includes over 900 buses (250 trolleys and over 650 diesel buses) as well as a ferry system (SeaBus) and light rapid transit (SkyTrain). The SeaBus, the first marine transit service of its kind in the world, connects the towns of North Vancouver and Vancouver, carrying about 2 million passengers per year. The SkyTrain system, the longest completely automated, driverless rapid transit system in the world began in 1985. It now links Vancouver with several eastern suburbs. The \$32 million SkyBridge over the Fraser River between New Westminster and Surrey is the longest rapid-transit-only bridge in the world.

4.1. Existing and candidate transit centre locations

The Vancouver Regional Transit System includes five transit centres. For this study, five additional locations suggested by BC Transit staff were considered. Table 1 lists each site, its current capacity and allocation (for existing sites), the minimum and maximum capacities used in the mixed integer program and whether or not it can house trolley buses. Sites indicated with an "E" or "C" are existing or candidate locations respectively. Figure 1 shows the 10 sites.

4.2. Model operation

The final model consisted of 12,192 continuous assignment variables (X_{rdps}). These are determined in the following manner: 98 diesel routes * 3 days * 4 service periods * 10 locations gives 11,760 variables. For the trolley routes, there are 12 routes * 3 days * 4 service periods * 3 locations for 432 variables. As explained previously, there are 31 integer variables (20 general and 11 binary).

A matrix generator was created to format the information for input to *CPLEX 2.0*, a state-of-the-art commercial mixed integer programming code. All model runs were made on a HP 9000 series 700 model 730 workstation. This workstation employed a 66 MHz PA-RISC processor supporting the HP-UX UNIX operating system. The processor was capable of 23.7 MFLOPS in floating point operation.

CPLEX permits the operator to solve the mixed integer programming problem by using one of two node search strategies, best-bound or depth-first. A best-bound strategy uses a search procedure that spends considerable time "hopping around" from branch to branch near the top of a branch-and-bound tree, computing the resulting values of decision nodes. On the other hand, depth-first search dives deep into a tree, enumerating all possible values along its way. The advantage to adopting a best-bound strategy is that, unlike depth-first search, one will not waste valuable computer time evaluating a large number of nodes in a rather poor branch. However, best-bound searches can eat up considerable time hopping from branch to branch near the top of the tree.

The use of these two search procedures for the present case study failed to reveal any

Table 1. Existing and candidate transit centre locations

Site number	Location	Current Allocation	Current Capacity	Minimum Size	Maximum Size	Trolley Accessible
E1	North Vancouver	78	60	50	60	
E2	Port Coquitlam	114	250	50	250	
E3	Surrey	132	250	50	250	
E4	Burnaby	166	160	50	160	YES
E5	Oakridge	410	350	50	350	YES
C1	British Columbia Rapid Transit Corp. (BCRTC)	—	—	50	250	
C2	Cloverdale	—	—	50	250	
C3	Lougheed Park & Ride	—	—	50	250	
C4	Richmond	—	—	50	250	
C5	Main & Terminal	—	—	50	250	YES

Fig. 1. Location of key transit facilities.

significant differences between them. Solution times were about the same. Because the times required to solve this problem to optimality were relatively small (roughly 7 min), the issue as to which specific search strategy to use was not deemed crucial. CPLEX used best-bound search as its default search technique; consequently, this was the routine used in later model runs.

5. MODEL RESULTS

Table 2 shows the allocations at each transit centre under the current scheme, the optimal solution, and the "minimum 100" experiment that is explained later. Each transit centre is given a name corresponding to a nearby landmark or the city in which it is located.

Under the optimal solution, none of the existing sites are closed. However, their capacity restrictions are enforced. The excess of buses at North Vancouver, Burnaby, and Oakridge are allocated to other garages. Two new facilities are opened near the BCRTC (an existing BC Transit structure in the town of New Westminster) and at the intersection of Main Street and Terminal Avenue in Vancouver. Trolleys are allocated to the Oakridge and the Main and Terminal facilities.

The total annual cost under the current configuration is \$14.885 million. The optimal solution produced an annual savings of over \$560,000 (3.77%). Excluding the capital costs of two new bus garages, deadheading cost dropped by almost \$1.6 million (10.73%).

In the "minimum 100" experiment, the minimum size of a candidate transit centre was increased from 50 to 100 to see which (if any) candidate facilities would remain in the solution. Even though transit personnel indicated that a 50-bus facility was the minimum allowed, they felt that a more realistic scenario would stipulate a 100-bus minimum for a candidate garage. The experiment resulted in only one new facility being constructed, facility C1 near BCRTC. The trolley fleet was split between Oakridge and Burnaby. The cost of this solution was \$14.501 million, with deadhead cost accounting for \$13.268 million. Although total costs of this solution increased over the 50-bus minimum scenario, the actual deadhead costs *decreased*. This is due to the shifting of trolley buses to Burnaby. Substantial deadhead cost savings are realized by operating some of these routes from Burnaby. However, this savings brings with it increased capital costs (the electrification costs necessary to ensure that Burnaby can house trolleys). Table 3 presents a breakdown of the total costs for the various scenarios.

The optimal solution resulted in savings in both deadhead and total costs. However, driver relief cost was not considered. This is an important part of nonrevenue transportation costs. Data regarding specific driver relief points (should a route be allocated from alternative transit centre locations) was not available. In discussions with schedulers and in examination of driver relief points on current routes it was noted that relief points were often placed at major intersections close to existing transit centres. Because most transit centres are located on major streets, placing relief points at locations where the bus route

Table 2. Model results

Site Number	Location	Current Allocation	Optimal Allocation	"Minimum 100" Allocation
E1	North Vancouver	78	60	60
E2	Port Coquitlam	114	114	105
E3	Surrey	132	123	131
E4	Burnaby	166	160	160
E5	Oakridge	410	350	350
C1	BCRTC	—	50	100
C2	Cloverdale	—	0	0
C3	Lougheed Park & Ride	—	0	0
C4	Richmond	—	0	0
C5	Main & Terminal	—	50	0

Table 3. Breakdown of costs

Cost Category	Scenario		
	Current Allocation	Optimal Allocation	"Minimum 100" Allocation
Deadhead costs	\$14,885,587	\$13,288,690	\$13,268,812
Capital costs	0	\$1,035,400	\$1,232,675
Total costs	\$14,885,587	\$14,324,090	\$14,501,487

crosses that major street is both logical and convenient. However, because candidate transit centres are on different major streets, using existing relief points in a mathematical programming model would tend to keep existing transit centres in the solution.

Therefore, a separate post-optimization assessment was made. Thirty-six routes changed transit centres between the current and optimal configurations. The middle of each of these routes was determined and a straight-line distance measured between this point and each competing transit centre. This distance was taken as a rough approximation of distance to a relief point. (It is important to note that for most existing relief points, particularly those on the same major street as the existing transit centre, a straight line was an excellent approximation for the distance to the relief point.) In the case where the buses of a route were split between transit centres, a weighted combination of the distances was computed, reflecting the proportions of *all-day* vehicles assigned to the depots.

Considering average vehicle speed, driver salaries, and the number of reliefs on each route, the driver relief distances were transformed into costs.

Relief costs fell about 15% for the 36 routes which changed transit centre location. Although this admittedly is a rough approximation, it does give a strong indication that driver relief will not erode potential savings in total costs.

6. SUMMARY AND CONCLUSIONS

A mixed integer programming model has been developed to determine the number, location, and the allocation of buses for transit centres for the Vancouver Regional Transit System. An overall annual savings of roughly \$560,000 was observed. At a discount rate of 8%, this would have a net present value of nearly \$6 million.

Once basic data was obtained and analyzed and once a matrix generator program was created, experiments with this system could be performed rapidly and at very low cost. The clarity of the mixed integer programming format and the availability of the powerful CPLEX mixed integer programming code on a workstation are strong arguments for this approach. We do note, however, that solution time in general is heavily tied to the number of integer variables in the problem formulation.

There is also one weakness in the optimal solution under the current formulation as was previously mentioned. For about 20% of the routes, buses for a given route are split between transit centres. For example, 25 buses on route #9 are at the Oakridge Transit Centre, while the other 11 are housed at the Main and Terminal Transit Centre. Although some route splitting currently occurs, scheduling personnel indicated that this is generally not desirable due to managerial inconvenience. If the model were modified to prevent splits, this would be likely to increase the number of integer variables required and increase solution time.

BC Transit has adopted this mathematical programming approach, given the acronym BUBLS (BUs Barn Location System), as a capital projects planning tool. Of particular interest is the employment of BUBLS to help determine the appropriate number, location, and size of transit centres if an advanced light rapid transit extension is built to the municipality of Richmond (see Map 1). From the standpoint of buses, the light rapid transit extension would transform existing Richmond bus routes from "spoke" routes directed toward the city center of Vancouver into short shuttle routes to rapid transit

stations within Richmond. For equivalent service, fewer buses would be needed and the inclusion of a transit centre in Richmond would be more attractive.

Acknowledgements—We thank the following employees of BC Transit: Robert N. Tribe, Vice-President Capital Projects and Engineering; W. H. (Bill) Green, Schedule Supervisor and Technical Assistant; Garry D. Andrews, Project Manager.

REFERENCES

- Atkins, D. (1992). Personal communication.
- Balinski, M. L. (1961). Fixed-cost transportation problems. *NRLQ*, 8, 41–54.
- Bent, D. H. (1972). Branch-and-bound for facility location. *Infor*, 10, 1–7.
- Cooper, L. (1963). Location-allocation problems. *Ops Res.*, 11, 331–343.
- Khumawala, B. M. (1974). An efficient heuristic procedure for the capacitated warehouse location problem. *NRLO*, 21, 609–623.
- Love, R. F. et al. (1988). *Facilities Location: Models and Methods*. New York: North-Holland.
- Maze, T. H. et al. (1982). Optimization methodology for bus garage locations. *Transportation Engg JI of ASCE*, 108, 550–569.
- Maze, T. H. et al. (1983). Application of a bus garage location and sizing optimization. *Transportation Research*, 17A, 65–72.
- Willoughby, K. A. (1993). *BUBLS: A mixed integer program for transit centre location in the lower mainland*. Masters of Science Thesis, The University of British Columbia.